**Linear RG equations**

**Aside on solving discrete RG equations**

So in general, we’d get an equations for the renormalized coupling constants like,



These are difference equations, and we can see how they could lead into differential equations, like the RG differential equations. Now note that what we’re chiefly interested in is the solution of these equations for K(n) → K(b), j(n) → j(b). The β function definition isn’t something that we need so much as the actual b dependence, although we should keep in mind its definition nonetheless. If there is another way to do this than below, then we’d proceed thusly. So this case below is just one possibility, though a frequent one. For now, we’d look for fixed points.



which implies that upon iteration, we get exactly the same result, which is the signature of criticality. Then we want to linearize the coupling equations around the fixed points since we’re interested in the behavior chiefly around the critical region. Note that if the critical region were at  then we’d have to perform a slightly different procedure here.



Now we recognize that the first term on the right hand side is simply the fixed point itself, so we get,



If we define,



then we can write this as,



And these linear difference equations we should be able to solve exactly using a matrix method I suppose. We’d just need to diagonalize the matrix, then we’d have two uncoupled equations to solve, and then convert back to the original basis.



And we can write this in matrix form as:



and so,



and finally



and now we want to put this in terms of b.







It is these exponents which we need. Note that if one exponent is negative and the other positive, then our solutions will go to:



and our variables have an unambiguous single parameter dependence. And now we’ll analyze particular models…